Longitudinal Dispersion in a Packed Bed

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McHenry and Wilhelm (*2*) recently reported measurements of axial diffusivity in packed beds and found good agreement between their results and those predicted on the basis of a stirred-tank model. The purpose of this note is to suggest an alternate model which gives a somewhat more detailed picture of the mechanism of longitudinal mixing. The proposed model was suggested by the work of G. I. Taylor and is, in fact, a generalization of Taylor's result (*4*) for an unpacked pipe as applied to the more complex geometry of a packed bed.

Taylor's analysis shows that the phenomenon of longitudinal dispersion is a consequence of radial mixing in a shear flow. In a packed bed radial mixing is a result of side-stepping, which occurs whenever a fluid element encounters a particle of packing and, in order to pass it, is forced to make a radial detour. In making a radial side-step the fluid element, in general, also goes to a region having a different axial velocity. Since the two necessary conditions for longitudinal dispersion, viz., radial diffusion and radial velocity gradients (within a void space), both exist in packed beds, longitudinal dispersion in such systems is to be expected.

A simple, generalized, mixing-length model for longitudinal dispersion can be developed by considering the observations made by an observer traveling with the mean flow velocity V_0. These observations are shown schematically in Figure 1. At a certain instant two events are observed:

1. Fluid element A, having coordinates $(r_0 z_{-1})$ and traveling at the mean velocity V_0, encounters a solid particle and, in order to by-pass it, side-steps to position r_1, where its velocity is changed to V_1.
2. Fluid element B, having coordinates $(r_0 z_1)$ and traveling at the mean velocity V_0, encounters a solid particle and, in order to by-pass it, side-steps to position r_{-1}, where its velocity is changed to V_{-1}.

If the flowing fluid is a solution, with the solute concentration at z_1 given by c_1 and that at z_{-1} by c_{-1}, as a result of the events described above there is a net flux of solute across the plane which is moving with the average velocity V_0:

$$(V_1 - V_0)c_{-1} + (V_{-1} - V_0)c_1 = \text{net flux} \qquad (1)$$

The velocity increments in excess of the mean can be related to the velocity gradient within a pore by the relation

$$(V_1 - V_0) = -(V_{-1} - V_0) = l_r \frac{\partial V}{\partial r} \qquad (2)$$

where l_r is a radial scale of turbulence. This radial scale represents an average side-stepping distance.

Similarly, the concentrations at z_{-1} and z_1 are related by

$$c_1 - c_{-1} = l_z \frac{\partial c}{\partial z} \qquad (3)$$

where l_z is an axial scale of turbulence. This axial scale represents an average "by-passing" distance. Substitution of (2) and (3) into (1) gives

$$\text{net flux} = -l_z l_r \frac{\partial V}{\partial r}\frac{\partial c}{\partial z} = -E_z \frac{dc}{\partial z} \qquad (4)$$

The axial eddy diffusivity E_z is therefore given by

$$E_z = l_z l_r \left[\frac{\partial V}{\partial r}\right] \qquad (5)$$

where $[\partial V/\partial r]$ is a velocity gradient which is characteristic of flow through the void spaces of a packed bed.

An approximate numerical value of the eddy diffusivity in a packed bed can be obtained by substituting into Equation (5) the results of recent concentration fluctuation studies (*3*) and by assuming a reasonable value for the characteristic velocity gradient. Concentration fluctuation data in packed beds showed that $l_r = \frac{1}{4} d_p$ and, through an approximate calculation, that $l_z = 7 l_r$. Assignment of a numerical value for the characteristic velocity gradient is necessarily somewhat arbitrary. However, from dimensional considerations a reasonable estimate is

$$\frac{\partial V}{\partial r} \approx \frac{V}{d_p} \qquad (6)$$

Substitution in Equation (5) gives an approximate numerical value for the axial Peclet group. Rounded to the nearest integer the result is

$$Pe_z \approx \frac{V d_p}{E_z} \approx 2 \qquad (7)$$

The purpose in presenting this mixing-length model is not to compute a theoretical axial Peclet group but rather to provide some insight into the fundamental difference between axial and radial mixing. The numerical value of the Peclet group calculated above is of very limited significance

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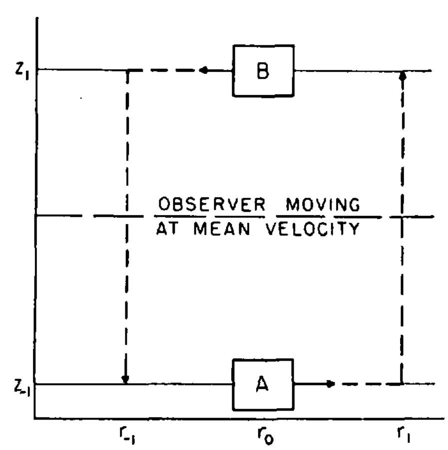

Fig. 1. Mechanism for longitudinal dispersion.

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because of the uncertainties in the approximations involved; the very good agreement with experimental results is probably fortuitous. However, even if somewhat different approximations had been used, the order of magnitude would still have been correct.

An interesting consequence of the mixing-length model is that it provides a qualitative prediction of the effect of void fraction on the axial Peclet group. As the void fraction rises, the bed becomes more loosely packed. The mixing-length model suggests that this loosening is accompanied by a small decrease in the velocity gradient and a slight increase in radial scale l_r. To a first approximation these effects will tend to cancel one another. However, the axial scale l_z, which is a measure of the by-passing length, would be expected to increase considerably. As a result the model predicts that an increase in void fraction produces a larger axial eddy diffusivity or a smaller Peclet group. Current experimental work (*1*) at Berkeley confirms this result.

ACKNOWLEDGMENT

The author is grateful to Theodore Vermeulen for helpful discussions.

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